Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 40779

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

First Semester

Civil Engineering

MA 8151 — ENGINEERING MATHEMATICS – I

(Common to: Aeronautical Engineering/Aerospace Engineering/Agriculture Engineering/Automobile Engineering/Biomedical Engineering/Computer Science and Engineering/Computer and Communication Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Electronics and Telecommunication Engineering/Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/Artificial Intelligence and Data Science/Bio Technology/Biotechnology and Biochemical Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Computer Science and Business System/Fashion Technology/Food Technology/Handloom and Textile Technology/Information Technology/Petrochemical Technology/Petroleum Engineering/Pharmaceutical Technology/Plastic Technology/Polymer Technology/Textile Chemistry/Textile Technology)

(Regulations 2017)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find $\lim\limits_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$.

2. Given $f(x) = \sin(\tan 2x)$, find $f'(x)$.

3. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z = x^2 y - x \sin xy$.

4. If $z = u^2 + v^2$ where $u = at^2$ and $v = 2at$, find $\frac{dz}{dt}$.

5. Evaluate $\int_1^9 \frac{x-1}{\sqrt{x}} dx$.

6. Determine whether the integral, $\int_0^\infty \frac{e^x}{e^{2x}+3} dx$ is convergent or divergent.

7. Evaluate $\int_1^2 \int_1^3 xy^2 dxdy$.

8. Evaluate $\iint_R r^2 \sin\theta dr d\theta$ where R is the region bounded by the semi-circle $r = 2a\cos\theta$ above the initial line.

9. Find the particular integral of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = 8\sin 2x$.

10. Reduce the differential equation given by $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log(x)\sin[\log(x)]$ to the one with constant coefficients.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the derivative of $f(x) = \frac{1-2x}{3+x}$ using the limit definition of the derivative. (8)

(ii) Let $g(x) = \begin{cases} x^2 + x, & x < 1 \\ a, & x = 1 \\ 3x + 5, & x > 1 \end{cases}$. Is there a value of a for which g is continuous at 1? If yes, find the same. Else give reason. (8)

Or

(b) (i) Find $\frac{dy}{dx}$ if $y = (x^3 - x + 1)^4 + \sqrt{x^2 + 1}$. (4)

(ii) Find the intervals on which the function $f(x) = x^4 - 2x^2 + 3$ is increasing or decreasing. Also find the local maximum and minimum values of $f(x)$. Find the intervals of concavity and the point of inflexion. (12)

12. (a) (i) If $u(x, y) = \tan^{-1} \frac{x^3 + y^3}{x + y}$, using Euler's theorem find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$. (8)

(ii) Expand $e^x \log(1 + y)$ in powers of x and y upto terms of third degree using Taylor's theorem. (8)

Or

(b) (i) If $x = uv$ and $y = \frac{u}{v}$, find $J = \frac{\partial(x, y)}{\partial(u, v)}$ and $J' = \frac{\partial(u, v)}{\partial(x, y)}$. Also verify $JJ' = 1$. (8)

(ii) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 sq. cm. (8)

13. (a) (i) Evaluate $\int e^{2x} \cos 3x dx$ using integration by parts. (8)

(ii) Using partial fractions method, evaluate $\int \frac{dx}{x^2 - 2x + 3}$. (8)

Or

(b) (i) Evaluate $\int \frac{1}{x^2\sqrt{x^2 + 4}} dx$ by substituting $x = 2\tan\theta$. (8)

(ii) Evaluate $\int \frac{x}{\sqrt{x^2 + x + 1}} dx$. (8)

14. (a) (i) Change the order of integration and hence evaluate $\int_0^a \int_y^a \frac{x dx dy}{x^2 + y^2}$. (8)

(ii) Find by triple integral, the volume of the tetrahedron bounded by the coordinate planes $x = 0$, $y = 0$, $z = 0$ and the plane $x + y + z = 1$. (8)

Or

(b) (i) Using double integrals, find the area between the parabola $x^2 = y$ and the line $x + y = 2$. (8)

(ii) Evaluate the triple integral, $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dx dy dz$. (8)

15. (a) (i) Use the method of variation of parameters to solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. (8)

(ii) Solve $x^2y'' + xy' + y = x\log(x)$. (8)

Or

(b) (i) Use the method of undetermined coefficients to find the complete solution of $\frac{d^2y}{dx^2} + 9y = \cos 3x$. (8)

(ii) Solve the simultaneous differential equations given by $\frac{dx}{dt} + 5x - 2y = t$, $\frac{dy}{dt} + 2x + y = 0$. (8)
